Sovereign Defaults and Banking Crises*

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Abstract

Sovereign defaults and banking crises have characterized the history of Emerging Economies. Three features of these crises are noteworthy: (i) Default and Banking crises tend to happen together, (ii) the banking sector is highly exposed to government debt, and (iii) crises episodes are episodes of decreased output and credit.

In this paper I provide a rationale for this phenomena: I propose a model economy that features facts (i) and (ii) and delivers fact (iii) endogenously. Financial intermediaries who are exposed to government debt suffer from a sovereign default that reduces the value of their assets. This forces the financial intermediaries to decrease the credit to the productive private sector. This credit crunch generates an endogenous output decline. The main results can be summarized as follows: reasonable parameterizations of the model deliver (1) Default in equilibrium, (2) v-shaped behavior of output around default/banking crises episodes, (3) mean output decline in default episodes = 4.5%, and (4) overall qualitative behavior of the model is in line with the data for emerging economies.

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1 Introduction

Sovereign defaults and banking crises have characterized the history of Emerging Economies. Three features of these crises are noteworthy:

i. Default and Banking crises tend to happen together.

ii. The banking sector is highly exposed to government debt.

iii. Crises episodes are episodes of decreased output and credit.

In this paper I provide a rationale for this phenomena: I propose a model economy that features facts (i) and (ii) and can deliver fact (iii) endogenously. Financial intermediaries are exposed to government debt and therefore suffer from a sovereign default that reduces the value of their assets. This forces the financial intermediaries to decrease the credit to the productive private sector. This credit crunch generates an endogenous output decline.

These dynamics that characterize a default and a banking crisis are obtained as the optimal response of a benevolent planner: faced with a level of spending that needs to be financed, and having only two instruments at hand (i.e. debt and taxes), the planner may find it optimal to default on its debt even at the expense of decreased output and consumption. The planner balances the costs and benefits of a default: the benefit is the lower taxation needed to finance the spending (lower than would otherwise have been necessary), the cost is the reduced credit availability and the consequently decreased output.

At the heart of the quantitative models of sovereign defaults lies a trade-off between costs and benefits of defaulting. The vast majority of this literature (noteworthy exceptions are discussed later on) has focused only on external debt and has assumed exogenous cost-of-default-structures. By doing this the benefit of a default is a transfer of resources from foreigners (holders of the defaulted debt) to domestic residents, and the cost is an (exogenously determined) output loss sometimes coupled with a reputational loss.
This paper provides a sovereign default framework where the debt is entirely domestic and nonetheless default incentives emerge (with endogenously determined output and credit contractions as the costs of default).

The main results can be summarized as follows: reasonable parameterizations of the model deliver:

1. Default in equilibrium,
2. V-shaped behavior of output around default/banking crises episodes,
3. Mean output decline in default episodes = 4.5%,
4. Overall qualitative behavior of the model is in line with the data.

The remainder of the paper is structured as follows. The next subsection summarizes the related literature. Section 2 presents stylized facts on sovereign defaults, banking crises and output drops. Section 3 lays out the model and defines the equilibrium. Section 4 contains the numerical solution and the main results of the paper. Section 5 concludes.

1.1 Related Literature

Quantitative models of sovereign default have received increased attention since the contributions of Aguiar and Gopinath (2006) and Arellano (2008). These papers extended the seminal framework of Eaton and Gersovitz (1981) to account for business cycle regularities in emerging economies. A large literature has emerged following this approach and several interesting aspects of the dynamics of sovereign debt have been studied recently.

The optimal default decision generated by this literature comes from weighing the costs and benefits of doing so. The vast majority of the literature has assumed an exogenous

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cost-of-default structure. My paper proposes a channel to endogenize these costs via the role of government debt in the domestic credit market.

Mendoza and Yue (2008) were the first (to my knowledge) to introduce endogenous costs of default. They assume that a sovereign default not only excludes the government from the international markets but also prevents the private sector firms from tapping foreign markets. In this way, a sovereign default ‘forces’ the productive sector to use less efficient resources and hence generates an output cost. My paper departs from Mendoza and Yue (2008) in two relevant ways: first, it presents a closed economy set-up that makes default less attractive (there are no resources transferred from abroad in the event of default) and therefore makes default on equilibrium a more challenging outcome; second, it acknowledges the high prevalence of government debt in financial intermediaries balance sheets and illustrate how a sovereign default diminishes credit availability in the economy.

Other researchers have noticed the exposure of the domestic banking sector to government debt and have asked different questions about this phenomenon. Gennaioli et. al. (2010) construct a model of domestic and external sovereign debt where domestic debt weakens the balance sheet of banks. This potential damage suffered by the banking sector represents in itself a ‘signaling’ device that attracts more foreign lenders\(^2\). My paper relates to Gennaioli et. al. (2010) in that it also develops a domestic market for government debt but it abstracts from an external market. On the contrary, I analyze a scenario in which default may be optimal even if 100% of the debt is domestically held.

Finally, my paper is also related to the optimal taxation literature. The closest paper in this literature is Pouzo (2009). He builds on the work of Aiyagari et. al. (2002) to analyze the optimal taxation problem of a planner in a closed economy with defaultable debt. My paper differs from Pouzo (2009) in three crucial aspects: first, Pouzo (2009) relies on an exogenous cost of default whereas this paper proposes an endogenous structure; second,

\(^2\) Basu (2009) develops a similar model that features domestic and foreign creditors and where domestic economic fragility allows the sovereign to borrow from international markets. Alessandro (2009) develops a related model in which the sovereign default increases the borrowing costs to domestic firms.
Pouzo (2009) focuses on the dynamics of the optimal tax policy and my focus is more on the output dynamics around defaults; and third (on a more technical note), Pouzo (2009) solves an equilibrium in which the government has commitment to a certain tax schedule but not to a repayment policy, whereas my paper assumes no commitment whatsoever on the side of the government.\(^3\)

2 Stylized Facts

In this section I document the stylized facts that motivate the theoretical/quantitative analysis presented in the rest of the paper. I begin by describing the time clustering of default crises and banking crises. Then, I examine the exposure of the emerging economies banking sectors to government debt. Finally, I present the output and credit behavior around default episodes.

Default and Banking crises tend to happen together. In my theory, a sovereign default renders the banking sector unable to extend credit to the private sector by decreasing the value of its assets: this is what I call a banking crisis.\(^4\)

To document the time profile of defaults and banking crises I follow Reinhart and Rogoff (2009) classification of banking crises to identify those crises that occur in the temporal vicinity of a sovereign default. Of the 82 banking crises episodes documented in Reinhart (2010), 70 were coupled with default crises. From those 70 episodes I only consider crises after 1970 (due to data limitations) and I identify those in which the sovereign default either preceded or coincided with the banking crises: Table 1 shows this accounting exercise. I

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\(^3\)This paper is also related to the literature on optimal public policy without commitment. These papers solve for Markov-Perfect (and therefore, time-consistent) optimal policy. See Klein et. al. 2008 and references therein.

\(^4\)Demirguc-Kunt and Detragiache (1998) define as a banking crisis any episode in which at least one of the following criteria is true: (1) The ratio of non-performing assets to total assets in the banking system exceeds 10 percent, (2) The cost of rescue operation was at least 2 percent of GDP, (3) Banking sector problems resulted in a large scale nationalization of banks, (4) Extensive bank runs took place or emergency measures (e.g. deposit freezes, prolonged bank holidays, generalized deposit guarantees) were enacted by the government. The mechanism highlighted in the paper is closely related to (1).
find that approximately 64% of the crises episodes are consistent with this paper’s assumed timing of events.

Table 1. Timing of Defaults and Banking Crises

<table>
<thead>
<tr>
<th>Banking Crises in:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$t - 2 \text{ or } t - 1$</td>
<td>14 (36%)</td>
</tr>
<tr>
<td>$t$</td>
<td>13 (33%)</td>
</tr>
<tr>
<td>$t + 1 \text{ or } t + 2$</td>
<td>12 (31%)</td>
</tr>
</tbody>
</table>

The relationship between Banking crises and Default episodes has been previously studied in the empirical literature. In particular, the question of whether a default causes a banking crises or vice versa has been recently studied by Borensztein and Panizza (2008). They construct an index of banking crises that includes 149 countries for the period 1975-2000. In this sample they identify 111 banking crises (implying an unconditional probability of having a crisis equal to 2.9%) and 85 default episodes (unconditional default probability of 2.2%). Their results are summarized in the following table:

Table 2. Probabilities of Default and Banking Crises (from Borensztein and Panizza (2008))

<table>
<thead>
<tr>
<th>Probabilities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Unconditional Prob. of a banking crisis</td>
<td>2.9</td>
</tr>
<tr>
<td>Prob. of banking crisis conditional on default</td>
<td>14.1</td>
</tr>
<tr>
<td>p-value on the test: $\text{prob}(bc/def) &gt; \text{prob}(bc)$</td>
<td>0.0</td>
</tr>
<tr>
<td>Unconditional Prob. of a sovereign default</td>
<td>2.2</td>
</tr>
<tr>
<td>Prob. of default conditional on banking crisis</td>
<td>4.5</td>
</tr>
<tr>
<td>p-value on the test: $\text{prob}(def/bc) &gt; \text{prob}(def)$</td>
<td>0.1</td>
</tr>
</tbody>
</table>

When conditioning on a sovereign default episode, the probability of a banking crisis increases by a factor of 5 and this conditional probability is statistically significant from the unconditional one (as denoted by a 0.0 p-value).
It can be argued that a banking crises can generate additional government spending (for example in the form of bailouts) that would leave the sovereign more prone to a default. It is then imperative to examine the probability of having a default conditional on experiencing a banking crisis: this probability is only 2 percentage points higher than the unconditional probability and it is not statistically significant (at either the 1% and the 5% confidence levels) from the unconditional one.

Even tough this results should be taken with a grain of salt, they suggest that a default may increase the probability of a banking crisis much more then the other way around. Overall, the evidence presented provides support for the assumed timing in the model.

**The banking sector exposure to government debt.** The central point of this paper is that a sovereign default damages the lending ability of banks due to the amount of defaultable paper in their balance sheets. In this section I take a look at the degree of exposure of banks to government debt. To do this I define an exposure ratio in the following way:

\[
\text{Financial Institutions’ net credit to the gov’t} \quad \frac{\text{Financial Institutions’ net total assets}}{}
\]
Figure 2: Costly Defaults and Banking crises

As Figure 1 documents, this exposure ratio averages 30% for emerging economies. What is even more compelling is that for countries that actually defaulted (like Russia and Pakistan) this percentage was even higher.

Two additional pieces of information are relevant: first, this high exposure is not due to government financial repression (recent increases in the cost government financing are precisely due to a move from low interest rates to market-based interest rates, see Caprio (1999) and Reinhart and Rogoff (2008) for further evidence on this) and second, the typical capitalization ratio for domestic financial institutions is roughly 10% of the assets. This latter fact implies that a default would leave the banking sector in many emerging economies technically insolvent.

Crises episodes are episodes of decreased output and credit. It has being documented that output falls in the event of a sovereign default (see for example Sturzenegger and Zettelmeyer 2006). Figure 2 makes the same point in a visual way. We observe a v-shape behavior of GDP in the temporal vicinity of defaults, being this behavior more severe when defaults are coupled with banking crises.

\footnote{Between 2001 and 2002 in Argentina, considerations on the harm the banking system would take were in the front row of the discussions that eventually lead to the default decision. See Perry and Serven (2003) and Kumhof (2004).}
Default and Banking crises are also characterized by decreased private credit to domestic productive firms. To document this fact I use the Financial Structure Dataset constructed by Beck and Demirgüç-Kunt (2009) to look at the behavior of Private credit by deposit money banks around defaults and banking crises. Figure 3 plots this measure as a percentage of GDP and shows that when a default comes private credit shrinks and remains reduced for the subsequent periods.

To summarize the set of facts just reviewed: 1- Default and Banking crises tend to happen together (with 64% of banking crises happening together with or right after a default), 2- the banking sector is highly exposed to government debt (with emerging economies banking sector holding on average 30% of their assets in government bonds), and 3- crises episodes are episodes of decreased output and credit.

3 Environment

I analyze a closed economy under discrete time, $t = 0, 1, 2, ...$ There are four players in this economy: households, firms, financial intermediaries (or bankers), and a (benevolent) government.
In this framework the households do not have any intertemporal choice so they only make two decisions: how much to consume and how much to work (i.e., this is just a consumption/leisure problem from the households’ point of view). The production in the economy is conducted by standard neoclassical firms that face only a working capital constraint: they need to pay a fraction of the wage bill up-front, hence their need for external financing.

The bankers are the ones in charge of the intermediation (from the funds available to them, they lend to the firms and the government). These bankers start the period with two assets: $A$ and $b$. $A$ represents an endowment that the bankers receive every period\(^6\). $b$ represents the level of sovereign debt owned by the bank at the beginning of the period.

Finally, the government is a benevolent one (i.e., it tries to maximize the residents’ utility). It faces a stream of spending that must be financed and it has three instruments to do so: labor income taxation, borrowing, and default. I do not assume any kind of commitment technology available to the government: this means that every period the government can walk away on its debt. This default decision is taken at the beginning of the period and influences the rest of the economic decisions. Therefore the following subsections examine how this economy works under both default and no-default, and ultimately how the sovereign optimally chooses its tax, debt, and default policies.

### 3.1 Decision problems

In this section we describe the problems faced by each of the four economic agents in the economy. The variable $d$ stands for the default decision and can take only two values: 0 (no default) or 1 (default).

\(^6\)There are a number of ways to understand/model this endowment $A$. One alternative is to model deposits dynamics, then $A$ is composed of fresh deposits received by the bankers in period $t$ (more on the desirability of modelling deposit dynamics later on). An alternative interpretation is that bankers are part of the household, and they are the only ones capable of conducting financial transactions, then it is optimal for the household (at the beginning of the period) to give this endowment $A$ to the members of the household that will use it better: the bankers.
3.1.1 Household’s problem

As indicated above, the only decisions are the labor supply and consumption levels. Therefore the problem faced by the households can be expressed as:

\[
V(b_t, z_t) = \max_{\{c_t, n_t\}} \sum \beta^t U(c_t, n_t) \tag{1}
\]

s.t. \( c_t = (1 - \tau_t)w_t n_t + \Pi_t^F \tag{2} \)

where \( c_t \) stands for consumption, \( n_t \) denotes labor supply, \( w_t \) is the wage rate, \( \tau_t \) is the labor-income tax rate, and \( \Pi_t^F \) represents the firm’s profits.

Plugging equation (2) into equation (1) the household’s problem can be rewritten as:

\[
V(b, z) = \max (n_t) \sum \beta^t U((1 - \tau_t)w_t n_t + \Pi_t^F, n_t) \]

and the period-t FOC reads as:

\[
U_c (1 - \tau_t)w_t + U_n = 0 \tag{3}
\]

Equation (3) can be rearranged into the familiar expression equating the marginal rate of substitution between leisure and consumption to the after-tax wage rate:

\[
- \frac{U_n}{U_c} = (1 - \tau_t)w_t \tag{4}
\]

Therefore, equations (2) and (4) are the optimality conditions that emerge from the household’s problem.

3.1.2 Firm’s problem

The firms in this economy demand labor to produce the consumption good. But the environment is not completely frictionless: they face a working capital constraint that requires
them to pay up-front a certain fraction of the wage bill. Given these features, the firm’s problem can be expressed as:

\[
\max_{N_t, l_t^d} \Pi_t^F = z_t F(N_t, K) - w_t N_t + l_t^d - (1 + r_t)l_t^d
\]  

(5)  

\text{s.t. } \gamma w_t N_t \leq l_t^d 

(6)  

where \(z\) is an aggregate technology shock, \(F(N, K)\) is the production function, \(l_t^d\) is the demand for working capital loans, \(r_t\) is the rate charged for the loan, and \(\gamma\) is the fraction of the wage bill that must be paid up-front.

The working capital constraint is captured by equation (6). This equation will always hold with equality. Taking this into account and plugging the constraint into the objective function we obtain:

\[
\max_{N_t} \Pi_t^F = z_t F(N_t, K) - (1 + \gamma r_t) w_t N_t 
\]

The period-t FOC is:

\[
z_t F_N = (1 + \gamma r_t) w_t 
\]

(7)  

Therefore, the optimality conditions from the firm’s problem are equation (7) and equation (6) evaluated with equality.

3.1.3 Banker’s problem

Bankers are in charge of the financial intermediation. Every period they face two different credit markets: the loans market and the sovereign bonds markets. The working assumption is that they participate in these markets sequentially\(^7\).

\(^7\)The assumption of sequential banking is no different from the day-market/ night-market or the decentralized-market/ centralized-market assumption commonly used in the money-search literature (see Lagos and Wright (2005)).
The problem of the banker can be written in recursive form as:

\[ W(b, z) = \max_{x, l_s, b'} v(x) + \delta E W(b', z') \tag{8} \]

s.t. \[ x = A + (1 - d)b + l_s r - (1 - d)q b' \tag{9} \]

\[ A + (1 - d)b \geq l_s \tag{10} \]

where \( l_s \) stands for working capital loans supply, \( b' \) represents government bonds demand, \( A \) is the bankers’ endowment, \( r \) is the interest rate on the working capital loans, and \( q \) is the price per sovereign bond.

\( v(x) \) is the period return function of the banker, \( x \) is the end-of-period consumption of the banker, \( \delta \) stands for the discount factor.

Equation (10) captures the implicit timing of the banking sector: the maximum the banker can lend to the firms is the sum of his endowment and the repayment of government debt.

This problem can be rewritten as:

\[ W(b, z) = \max_{l_s, b', \mu} v(A + (1 - d)b + l_s r - (1 - d)q b') + \delta E W(b', z') + \mu [A + (1 - d)b - l_s] \]

where the FOCs are:

\[ l_s : \quad v'(x)r - \mu = 0 \tag{11} \]

\[ b' : \quad -v'(x)q(1 - d) + \delta E W_b(b', z') = 0 \tag{12} \]

\[ \mu : \quad A + (1 - d)b - l_s \geq 0 \quad \& \quad \mu [A + (1 - d)b - l_s] = 0 \tag{13} \]

The envelope condition reads as:

\[ W_b(b, z) = v'(x)(1 - d) + \mu(1 - d) \]
therefore:

$$W_b(b', z') = v'(x')(1 - d') + \mu'(1 - d')$$  \hspace{1cm} (14)

Then, rearranging equation (11) we get:

$$r = \frac{\mu}{v'(x)}$$  \hspace{1cm} (15)

Combining equations (12), (14) and (15) we get:

$$q = \frac{\delta}{(1 - d)} E \left\{ \frac{v'(x')}{v'(x)} (1 - d')(1 + r') \right\}$$  \hspace{1cm} (16)

To better read this pricing equation, let’s relabel things. I define (as it is usual in macroeconomics) the Stochastic Discount Factor (SDF) of the banker as:

$$\delta E \left\{ \frac{v'(x')}{v'(x)} \right\} \equiv \Lambda_{t, t+1} = SDF$$  \hspace{1cm} (17)

I also summarize the future payoff of the sovereign bond as:

$$\vartheta_{t+1} = (1 - d')(1 + r')$$  \hspace{1cm} (18)

This last expression says that the bond payoff must be such that it compensates the lender (in this case the banker) for the expected default probability: in case of default the lender not only loses his investment in sovereign bonds but also loses the future gains that those bonds would have created had them been repaid. Those gains are summarized in $r'$.

Therefore we arrive at the familiar asset-pricing equation:

$$q_t = \frac{1}{(1 - d)} E_t \left\{ \Lambda_{t, t+1} \vartheta_{t+1} \right\}$$  \hspace{1cm} (19)

Equation (19) reflects how the exclusion after a default is captured by the price: in the
case of a default \( d = 1 \) and then \( q \to \infty \) therefore there is no trading of government debt.

### 3.1.4 Government Budget Constraint

The government needs to tax labor income to pay for both the exogenous spending and (in case it decides not to default) the debt obligations. Its budget constraint can be expressed as:

\[
g + (1 - d_t)B_t = \tau_t w_t n_t + (1 - d_t)B_{t+1}q_t
\]

where \( B_t \) stands for debt (with positive values meaning more debt), \( g \) is the exogenous government spending, \( \tau_t \) is the labor income tax-rate, \( w_t \) is the wage rate, and \( n_t \) stands for labor.

### 3.2 Competitive Equilibrium of the Private Sector

**Definition 1** A Competitive Equilibrium for the Private Sector of the Economy is a sequence of allocations \( \{c_t, x_t, n_t, N_t, l_{t}^{d}, l_{t}^{s}, b_{t+1}\}_{t=0}^{\infty} \) and prices \( \{r_t, w_t, \Pi_t\}_{t=0}^{\infty} \) such that given sovereign bond prices \( \{q_t\}_{t=0}^{\infty} \), government policies \( \{\tau_t, d_t, B_t\}_{t=0}^{\infty} \), and shocks \( \{g, z_t\}_{t=0}^{\infty} \) the following holds:

1. \( \{c_t, n_t\}_{t=0}^{\infty} \) solve the households’ problem.
2. \( \{N_t, l_{t}^{d}\}_{t=0}^{\infty} \) solve the firm’s problem.
3. \( \{x_t, l_{t}^{s}, b_{t+1}\}_{t=0}^{\infty} \) solve the banker’s problem.

### 3.3 Sovereign Government

The sovereign government in this economy solves a problem similar to a Ramsey planner problem. It has three policy tools to maximize the welfare of the domestic residents: taxation, debt and default; but it is subject to two constraints: (1) the allocations that emerge from
the government policies should represent a competitive equilibrium, and (2) the government budget constraint must hold.

The optimization problem of the government can be recursively written as:

$$V(b, z) = \max_{d \in \{0, 1\}} (1 - d)V^{nd} + dV^d$$ (21)

Given that there are two kind of residents (households and bankers) the overall objective function of the planner is a convex combination of the value functions of these two residents. Then:

$$V^i(b, z) = \theta V^i(b, z) + (1 - \theta)W^i(b, z)$$

where $i = d, nd$ and $\theta$ is the weight assigned to the households’ happiness in the planner’s objective function.

Therefore, the value of no default is:

$$V^{nd}(b, z) = \max_{n, c, b', x} \{\theta V^{nd}(b, z) + (1 - \theta)W^{nd}(b, z)\}$$

subject to:

$$V^{nd}(b, z) = U(c, n) + \beta EV^{nd}(b', z')$$
$$W^{nd}(b, z) = v(x) + \delta EW^{nd}(b', z')$$

$$g + b = \tau wn + b'q$$ (gov’t b.c.)
$$c + x + g = zF(K, n) + A$$ (resources const.)
$$x = (A + b)(1 + r) - qb'$$
$$r = \frac{znF}{b + A} - \frac{1}{\gamma}$$
$$-\frac{V_n}{V_c} = (1 - \tau)w$$ (comp. eq. conditions)
$$w = \frac{znF}{(1 + \gamma r)}$$
$$q = \delta E\left\{\frac{v'(x')}{v'(x)} (1 - d')(1 + r')\right\}$$

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If we clean things up and express the constraints in terms of allocations only:

\[ V^{nd}(b, z) = U(c, n) + \beta EV^{nd}(b', z') \]
\[ W^{nd}(b, z) = v(x) + \delta EW^{nd}(b', z') \]
\[ g + b = \frac{znF_N}{(1 + \gamma r^*)} + \frac{U_n(c, n)n}{U_c(c, n)} + b'q^* \]
\[ c + x + g = zF(K, n) + A \]
\[ x = (A + b)(1 + r^*) - b'q^* \]

where

\[ r^* = \frac{znF_N}{b + A} - \frac{1}{\gamma} \]
\[ q^* = \delta E \left\{ \frac{v'(x')}{v'(x)}(1 - d')(1 + r') \right\} \]

\( V^{nd} \) and \( W^{nd} \) represent the values of the household and the banker under no-default, respectively.

In case the sovereign decides to default it gets excluded from the credit market in that period. There is a probability \( \phi \) that the government will regain access to the financial market in which case its debt is forgiven (i.e. it gets a fresh start). Then, the value of default can be written as:

\[ V^d(z) = \max_{n,c,x} \left\{ \theta V^d(z) + (1 - \theta)W^d(z) \right\} \]

subject to:
\[ V^d(z) = U(c, n) + \beta E \left\{ \phi V^{nd}(0, z') + (1-\phi)V^d(z') \right\} \]
\[ W^d(z) = v(x) + \delta E \left\{ \phi W^{nd}(0, z') + (1-\phi)W^d(z') \right\} \]
\[ g = \tau wn \quad \text{(gov’t b.c.)} \]
\[ c + x + g = zF(K, n) + A \quad \text{(resources const.)} \]
\[ x = A(1+r) \]
\[ r = \frac{znF_N}{A} - \frac{1}{\gamma} \]
\[ \frac{U_n}{U_c} = (1-\tau)w \]
\[ w = \frac{zF_N}{(1+\gamma r)} \quad \text{(comp. eq. conditions)} \]

This problem can be re-expressed as:

\[ \mathcal{V}^d(z) = \max_{n,c} \left\{ \theta V^d(z) + (1-\theta)W^d(z) \right\} \]

subject to:

\[ V^d(z) = U(c, n) + \beta E \left\{ \phi V^{nd}(0, z') + (1-\phi)V^d(z') \right\} \]
\[ W^d(z) = v \left( znF_N - A \frac{1-\gamma}{\gamma} \right) + \beta E \left\{ \phi W^{nd}(0, z') + (1-\phi)W^d(z') \right\} \]
\[ zF(K, n) = c + znF_N + \frac{U_n(c, n)}{U_c(c, n)} \]

### 3.4 Recursive Competitive Equilibrium

**Definition 2** The Markov Perfect Equilibrium for this economy is (i) a borrowing rule \( b'(b, z) \) and a default rule \( d(b, z) \) with associated value functions \( \{ \mathcal{V}(b, z), \mathcal{V}^{nd}(b, z), \mathcal{V}^d(z) \} \), consumption(s) and labor plans \( \{ c(b, z), x(b, z), n(b, z) \} \) and taxation rule \( \tau(b, z) \), (ii) an equilibrium pricing function for the sovereign bond \( q(b', z) \), such that:

1. Given the price \( q(b', z) \), the borrowing and default rules solve the sovereign’s maximization problem in (21)
2. Given the price $q(b', z)$ and the borrowing and default rules; the consumption and labor plans $\{c(b, z), x(b, z), n(b, z)\}$ are consistent with competitive equilibrium.

3. Given the price $q(b', z)$ and the borrowing and default rules; the taxation rule $\tau(b, z)$ satisfies the government budget constraint.

4. The price equilibrium function satisfies equation (19)

4 Numerical Analysis

In this section I specify the functional forms, parameter values and stochastic processes used to solve the model numerically.

4.1 Functional Forms and Stochastic Processes

The period utility function of the households is:

$$U(c, n) = \frac{(c - \frac{\omega}{\omega})^{1-\sigma_c}}{1 - \sigma_c}$$

The return function of the bankers is:

$$v(x) = \frac{x^{1-\sigma_b}}{1 - \sigma_b}$$

The production function available to the firms is:

$$F(N, K) = N^\alpha K^{1-\alpha}$$

It is important to highlight that $K$ is fixed in my model, hence the production function is equivalent to $f(N) = \kappa N^\alpha$.

The only source of uncertainty in this economy is a productivity shock $z_t$. I model this
shock as following an AR(1) process:

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t$$

where $\varepsilon_t$ is an $i.i.d. N(0, \sigma^2_z)$.

### 4.2 Calibration

The model is calibrated at an annual frequency. Table 3 contains the parameter values used throughout the numerical solution.

<table>
<thead>
<tr>
<th>Table 3. Benchmark calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Curvature of labor disutility</td>
</tr>
<tr>
<td>Labor share in output</td>
</tr>
<tr>
<td>Household risk aversion</td>
</tr>
<tr>
<td>Banker risk aversion</td>
</tr>
<tr>
<td>Probability of redemption</td>
</tr>
<tr>
<td>Government Spending</td>
</tr>
<tr>
<td>Banker’s discount factor</td>
</tr>
<tr>
<td>Weight of hh. in planner’s obj. function</td>
</tr>
<tr>
<td>Household’s discount factor</td>
</tr>
<tr>
<td>Working capital requirement</td>
</tr>
<tr>
<td>Banker’s endowment</td>
</tr>
<tr>
<td>TFP autocorrelation coefficient</td>
</tr>
<tr>
<td>Std. dev. of innovations</td>
</tr>
</tbody>
</table>

The parameters above the line are set to independently match moments from the data or are parameters that take common values in the literature. The curvature of labor disutility is chosen to imply a Frisch wage elasticity of $1/(\omega - 1) = 2.2$ (as in Mendoza and Yue (2008)).
The labor share in output and the risk aversion parameter for the households $\sigma_c$ are set to 0.7 and 2 which are standard values in quantitative macroeconomics literature. Banker’s risk aversion $\sigma_b$ is set to zero to feature risk-neutral bankers. The probability of redemption is governed by the parameter $\phi$. Given the annual calibration and the evidence collected by Gelos et. al. (2008), this parameter takes a value of 0.25 in order to get a mean exclusion of 4 years. The value of the exogenous spending level $g$ is set up to 0.035 to match the ratio of general government consumption to GDP for Argentina in the period 1985-2000 of 5% (from World Bank’s World Development Indicators). The discount factor for the bankers (i.e., the lenders) takes a usual value in RBC models with annual frequency, 0.96. It is important to realize that the exact value of $\delta$ is not crucial in itself but in how it compares with the household’s discount factor (discussed below). Lastly, the weight that households’ utility receives on the planner overall objective function ($\theta$) is set in the benchmark calibration to 0.5 in order to give all the residents (bankers and households) the same weight. It is crucial to see that if $\theta$ takes the value of one, then the model features ‘international banks’ and hence facilitates the comparison with previous literature that focused on external debt and default: this is done as a robustness check later in the manuscript.

The parameters below the line $\{\beta, A, \gamma, \rho, \sigma_z\}$ are simultaneously determined in order to match a set of meaningful moments of the data. This is done by the Simulated Method of Moments (SMM) which consists of estimating the parameters by minimizing the distance between the moments generated by the model and those observed in the data.8

The moments matched by the model are all taken from Argentine data and they are the mean domestic debt to output, mean default rate, mean exposure of the banking sector to government debt, and the autocorrelation and standard deviation of GDP.

Given that the model economy is a closed one, the correct debt-to-output ratio to match is Domestic Debt to GDP. To do so I take the ratio of Total Debt to Output from Reinhart and Rogoff (2010) and extract only the Domestic Debt part of it by using the share of

8For a gentle introduction to SMM see Adda and Cooper (2003).
Domestic Debt to Total Debt form Reinhart and Rogoff (2008). This exercise gives a mean Domestic Debt to GDP ratio of 16.5% for the period 1980-2002.

According to the Reinhart and Rogoff (2010) Argentina has defaulted on its domestic debt 5 times since its independence in 1816, implying a default probability of 2.5%.

As it was documented in section 2 the banking sector of virtually every emerging economy is highly exposed to government debt. The mean exposure ratio (defined in section 2) for Argentina was approximately 25%.

The autocorrelation and standard deviation of Argentine GDP was computed at an annual frequency from the time series available from INDEC (the Argentine Census and Statistics Office) for the period 1980-2005. The autocorrelation of the cyclical component of GDP is 0.28 and the standard deviation is 4.11%.

4.3 Results

This section examines the performance of the benchmark calibration of the model in accounting for some key statistical moments of business cycles and government debt. Table 4 reports the moments from the simulations of the model and compares them with the moments from Argentine data.

---

9The series obtained from INDEC is real GDP, and it was first logged and then H-P filtered using a smoothing parameter of 6.25 (as suggested by Ravn and Uligh (2002)).
Table 4. Simulated Moments and Data

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma(c)/\sigma(y))</td>
<td>1.17</td>
<td>1.27</td>
</tr>
<tr>
<td>(\sigma(n)/\sigma(y))</td>
<td>0.57</td>
<td>0.97</td>
</tr>
<tr>
<td>(\text{corr}(\text{bond}_\text{spread}, y))</td>
<td>-0.62</td>
<td>-0.25</td>
</tr>
<tr>
<td>(\text{corr}(c, y))</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td>(\text{corr}(N, y))</td>
<td>0.52</td>
<td>0.97</td>
</tr>
<tr>
<td>(\text{corr}(\text{bond}_\text{spread}, N))</td>
<td>-0.58</td>
<td>-0.38</td>
</tr>
<tr>
<td>Default rate</td>
<td>2.56%</td>
<td>2.42%</td>
</tr>
<tr>
<td>Mean debt/ output</td>
<td>10.14%</td>
<td>9.32%</td>
</tr>
<tr>
<td>Mean output drop</td>
<td>9.40%</td>
<td>5.83%</td>
</tr>
<tr>
<td>Gov’t Spending/ output</td>
<td>5%</td>
<td>5.28%</td>
</tr>
<tr>
<td>Mean Exposure Ratio</td>
<td>36%</td>
<td>31.39%</td>
</tr>
</tbody>
</table>

Overall the benchmark calibration of the model does a very good job in replicating the data.

The model is able to account for various of the most salient facts of the Argentine economy\(^{10}\), namely a ratio of consumption volatility to output volatility greater than 1, countercyclical sovereign spreads, and a high and positive correlation between output and consumption. All of these moments were not targeted by the calibration process and are closely reproduced by the model.

The benchmark calibration also captures very nicely the qualitative behavior of the labor input. It is able to feature a negative correlation between employment and sovereign spreads, as well as procyclical employment.

\(^{10}\)These facts are by no means private property the Argentine economy. On the contrary they characterize many other emerging economies, as documented in Neumeyer and Perri (2005).
The model does a fair job when it comes to the mean output drop. Data from INDEC indicates that in the recent Argentine default episode GDP fell 10 percentage points from trend. The benchmark calibration delivers a decrease of 5%. The sovereign default triggers a credit crunch in the model economy and it in turns generates an output collapse. This collapse is due to a reduced access to the labor input, which is the only variable input in the economy. The fact that the economy cannot resort to a substitute input generates a sharper output decline. It is again important to remark that the mean output drop was not among the targeted moments in the calibration procedure.

The other moments in Table 4 are the targets used in the calibration and are reported for completeness and to show that the model does indeed approximate very well these statistics.

4.3.1 Macro dynamics around default episodes

In this subsection, I present a set of figures that document how three key macroeconomic variables behave around default events: output, private credit, and labor tax rates.

output’s v-shape behavior: the main result of the paper is to provide a framework
able to deliver endogenous output declines in default periods. Figure 4 is constructed from the model simulations in the following way: first, I identify the simulation periods where default happens; second, I construct a time series of 8 years before and 3 years after each default; third, I average across default episodes to construct a series of the mean output behavior around defaults.

As it is clear from Figure 4 the model features an output decline (and a consequent consumption decline) in the default period. But not only that, the model is also able to deliver a v-shape behavior of output around defaults: a strategic default is the optimal crisis resolution mechanism - - due to worsening economic conditions, the sovereign finds it optimal to default on its obligations instead of increasing the required taxation.

Figure 5 shows precisely this effect. In this figure I plot the equilibrium tax rates around defaults and also the 'counterfactual' tax rate that would have been necessary if instead of defaulting the government had repaid.

credit contraction: why does a default generate such a costly output decline? This paper proposes a credit crunch explanation: given that financial intermediaries hold gov-
ernment debt in their balance sheets, when a default comes a considerable fraction of their assets losses value, therefore their lending ability contracts\footnote{Bolton and Jeanne (2010) identify this phenomenon as banks becoming illiquid.} and as a consequence private sector credit diminishes. Given that the productive sector is in need of external financing, a credit crunch translates into an output decline.

Figure 6 shows how a credit crunch looks in my model. The benchmark parameterization of the model is able to produce a collapse in the private sector credit (i.e., working capital loans to firms, in the model).

Compared to the behavior observed in the data (i.e. Figure 2 in Section 2) the model is unable to generate any persistence. An interesting extension is to allow the bankers endowment $A$ to be endogenously chosen: in this way bad states of the nature (where default will sometimes happen) give little incentive to build-up $A$. When a default hits, then there is even less incentives to accumulate $A$ and this (possibly coupled with longer exclusion spells for the defaulting government) leads to lower private credit after a default.
Figure 7: Default Region

Figure 8: Price Schedule
4.3.2 Sovereign bonds market

Finally, I present two graphs typically found in papers of the sovereign debt literature: default regions and price schemes.

As discussed above, the model performs quite well with respect to the sovereign bond market dynamics: it produces default in bad times and (therefore) countercyclical spreads.

Figures 7 and 8 show the equilibrium default region and the bond price function, respectively. With respect to Figure 7, the white area represents the repayment area: it decreases (until disappearance) while the indebtedness level increases and/or the level of the technology shock decreases. Figure 8 presents the price schedule that the government faces. As it was expected, the price the sovereign receives for each bond decreases with TFP and also with the total level of indebtedness.

5 Conclusions

The prevalence of defaults and banking crises is a defining feature of emerging economies. These episodes are also characterized by: 1- Default and Banking crises tend to happen together (with 64% of banking crises happening together with or right after a default), 2- the banking sector is highly exposed to government debt (with emerging economies’ banking sector holding on average 30% of their assets in government bonds), and 3- crises episodes are episodes of decreased output and credit.

In this paper I have provided a rationale for this phenomena: I built a model economy that features facts (i) and (ii) and can deliver fact (iii) endogenously. Financial intermediaries who are exposed to government debt suffer from a sovereign default that reduces the value of their assets. This forces the financial intermediaries to decrease the credit to the productive private sector. This credit crunch generates an endogenous output decline.

The model showed an overall qualitative fit of the data (output and consumption drops, credit contractions at default). On a quantitative note, several dimensions are accurately
accounted for: countercyclicality of bond spreads, volatility of the spread, mean output drop. Other dimensions fall short from an accurate quantitative fit but have nonetheless the correct qualitative behavior: due to the closed economy and no-investment assumptions, the model is not able to generate higher volatility of consumption relative to output’s volatility, the model also exhibits a default frequency that is lower than the ones observed in the data.

Some avenues for future research / possible extensions are:

- Incorporating an endogenous banker’s net worth accumulation: this extension can in principle generate some persistence in the model and help to capture the time profile of domestic credit better.

- Allowing for both external and domestic debt: Having a closed economy makes defaults much less frequent than in an otherwise identical economy with access to international financial markets. Tapping world markets makes the sovereign more prone to default because such a repudiation implies a transfer of resources from abroad to the domestic economy. On the other hand, having the external market as well as the domestic one opens the door for analyzing very interesting dynamics like the optimal debt-portfolio composition of the government.
References


